PERTH MODERN SCHOOL
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Mathematics Specialist
Unit 3

TEST 2

Student name: Ann Swers

Teacher name: ____________________

Time allowed for this task: 50 minutes, in class, under test conditions
Calculator-Free

Materials required:

Standard items: Pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters, SCSA Formula Sheet.

Special items: Drawing instruments, templates

Marks available: 44 marks

Task weighting: **8%**

3

Question 1 **(~~2~~ marks)**

If $g(x) = \frac{\sqrt{x^2 - 1}}{x}$,find all solutions to:

(a) $g(\sqrt{2})$ $= \frac{\sqrt{(\sqrt{2})^2 - 1}}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$ ✓ or $\frac{\sqrt{2}}{2}$

(b) $g(0{\cdot}5)$

$= \frac{\sqrt{0.5^2 - 1}}{0.5}$ ✓ or $\frac{\sqrt{-\frac{3}{4}}}{\frac{1}{2}} = \frac{\sqrt{-3}}{\cancel{2}} \times \cancel{2}$

$\Rightarrow$ done if $x \in \mathbb{R}$ $= \sqrt{3}\,i$ ✓

Question 2 **(~~5~~ marks)** 4

State the domain and range of the following.

(a) $h(x) = \frac{1}{x + 1}$

$D_h : \{x : x \neq -1,\ x \in R\}$ ✓

$R_h : \{y : y \neq 0,\ y \in R\}$ ✓

(b) $m(x) = \sqrt{x^2 - 9}$

$D_m : \{x : x \leq -3,\ x \geq 3,\ x \in R\}$ ✓

$R_m : \{y : y \geq 0,\ y \in R\}$ ✓ -1 if not stated $x \in R$

Question 3

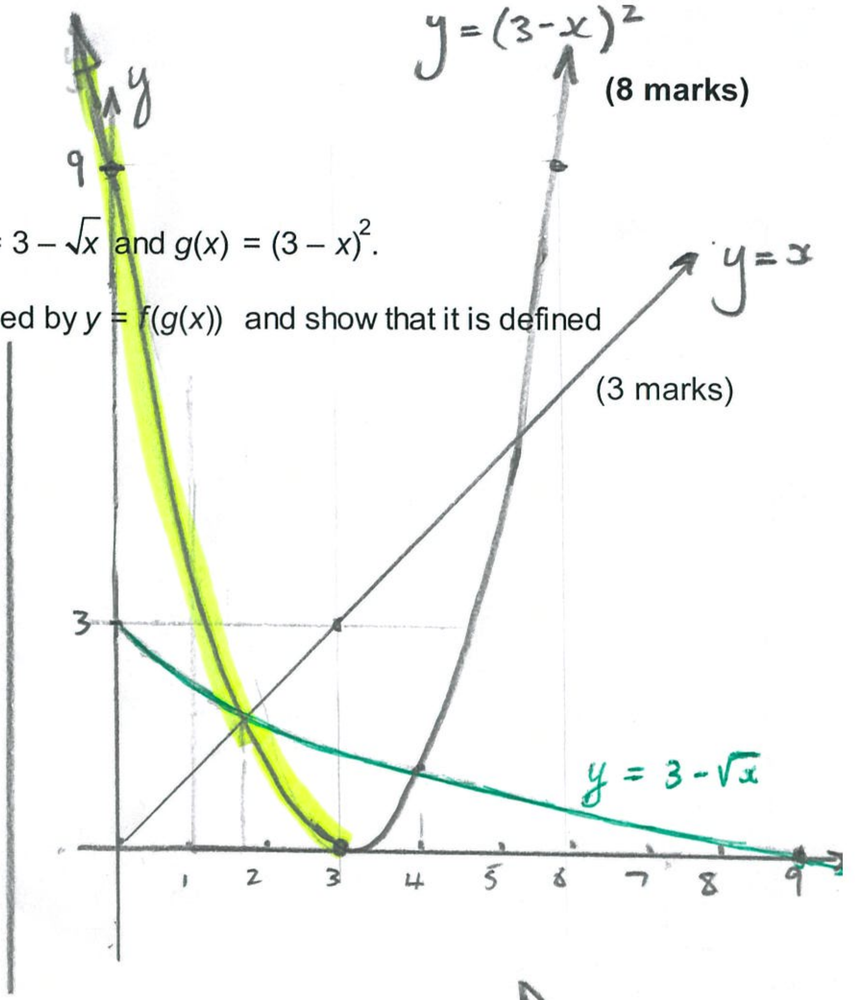
(8 marks)

R_g = All Reals ≥ 0
R_f = all Reals ≤ 3
$\therefore R_g = D_f$

The functions f and g are given by

$$f(x) = 3 - \sqrt{x} \text{ and } g(x) = (3 - x)^2.$$

(a) Determine the function defined by $y = f(g(x))$ and show that it is defined for all real values of x. (3 marks)



D_g = all Reals ; D_f = all Reals ≥ 0

Also accept

$f(3-x)^2$

$= \left[3 - \sqrt{(3-x)^2}\right]$

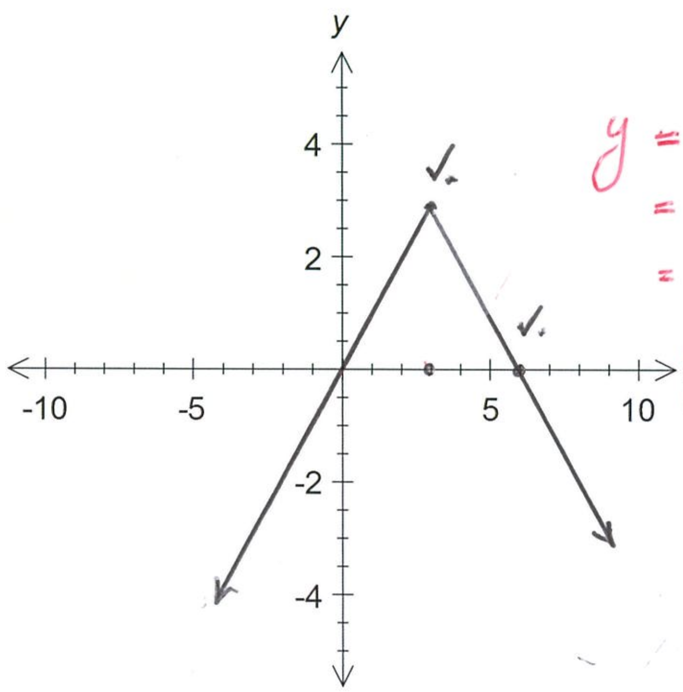
$= 3 - |3 - x|$ ✓.

$= \begin{cases} 3-(3-x), & x<3 \\ 3-(x-3), & x \geq 3 \end{cases}$

✓.

$= \begin{cases} x, & x<3 \\ 6-x, & x \geq 3 \end{cases}$ ✓.

(b) On the axes below, sketch the composite function $y = f(g(x))$. (2 marks)



$y = 3 - |3-x|$

$= -|3-x| + 3$

$= -|-(x-3)| + 3$

$\therefore y = -|x-3| + 3$

(c) How should the domain of $g(x)$ be changed so that $f(x)$ and $g(x)$ are inverse functions of each other? (3 marks)

$y = 3 - \sqrt{x}$

$\sqrt{x} = 3 - y$

$x = (3-y)^2$

$f(x) = 3 - \sqrt{x}$ for $x \geq 0$

$y \leq 3$

$f^{-1}(x) = (3-x)^2$ for $x \leq 3$

$y \geq 0$

✓

Hence domain of $g(x) = \{x : x \leq 3\}$

✓. States $g^{-1}(x)$ needs t[o] be one to one

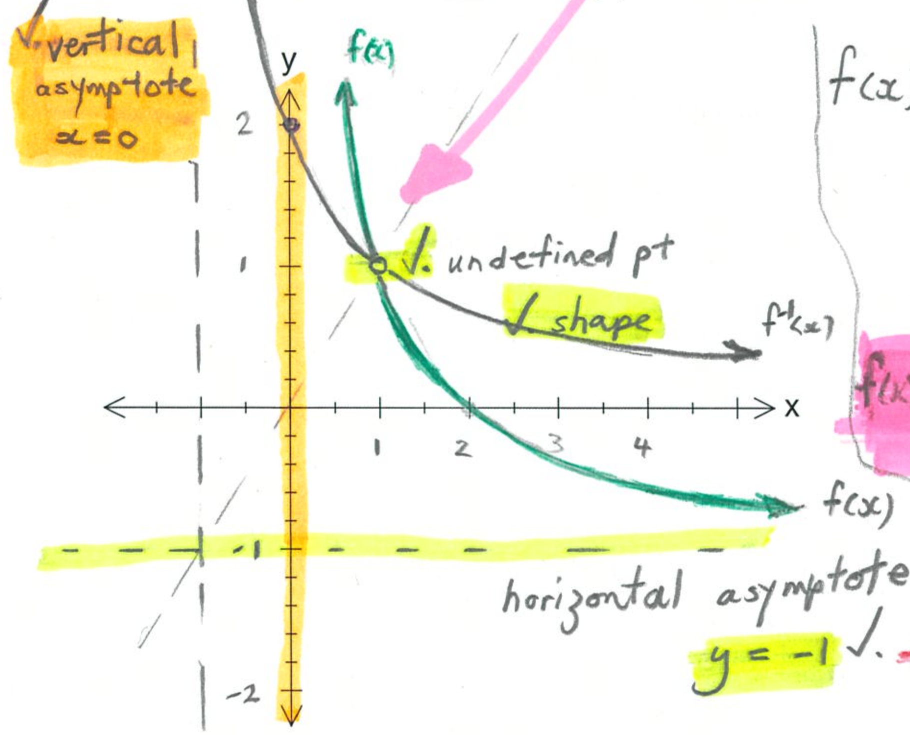
✓. States range of $f(x) \Rightarrow$ domain $g(x$

Question 4

(~~9~~ 10 marks)

The function $f(x)$ is defined for, $x > 0$ by $f(x) = \dfrac{-2 + 3x - x^2}{x^2 - x}$.

(a) Sketch the graph of $f(x)$ on the axes below. (4 marks)



$$f(x) = \frac{-1(x^2 - 3x + 2)}{x^2 - x}$$

$$= \frac{-1(x-2)\cancel{(x-1)}}{x\cancel{(x-1)}}$$

$$f(x) = \frac{2-x}{x}, \quad x - 1 \neq 0, \quad x \neq 1, \quad x > 0$$

$$\Rightarrow \frac{-\infty + 2}{\infty} = -1$$

(b) What is the range of $f(x)$? (~~1~~ 2 mark)

✓ $y > -1$ and ✓ $y \neq 1$

(c) Show that $f^{-1}(x) = \frac{2}{x+1}$, $x > -1$, $x \neq 1$, and state the domain of $f^{-1}(x)$.

(2 marks)

$y = \frac{2-x}{x}, x > 0, x \neq 1$

Inverse $x = \frac{2-y}{y}$

$xy = 2-y$

$xy + y = 2$

$y(x+1) = 2$

$y = \frac{2}{x+1}$ ✓

$\therefore f^{-1}(x) = \frac{2}{x+1}, \ x > -1, \ x \neq 1$ ✓

(d) Sketch the graph of $f^{-1}(x)$ on the ***same axes used for part (a)***. Label your sketch clearly.

(2marks)

Question 5 **(5 marks)**

If $m(x) = \frac{1}{x}$ and $n(x) = 2x + 3$, find the values of x for which $mon(x) = nom(x)$

$m \circ n(x) = m(2x+3)$

$= \frac{1}{2x+3}$ ✓

$n \circ m(x) = n\left(\frac{1}{x}\right)$

$= 2 \times \frac{1}{x} + 3$

$= \frac{2}{x} + 3$ ✓

Equal when $\frac{1}{2x+3} = \frac{2}{x} + 3 \Rightarrow 2x+3 = \frac{x}{2+3x} \Rightarrow (2x+3)(2+3x) = x$

$x = 2(2x+3) + 3(x)(2x+3)$ ✓

$x = 4x + 6 + 6x^2 + 9x$

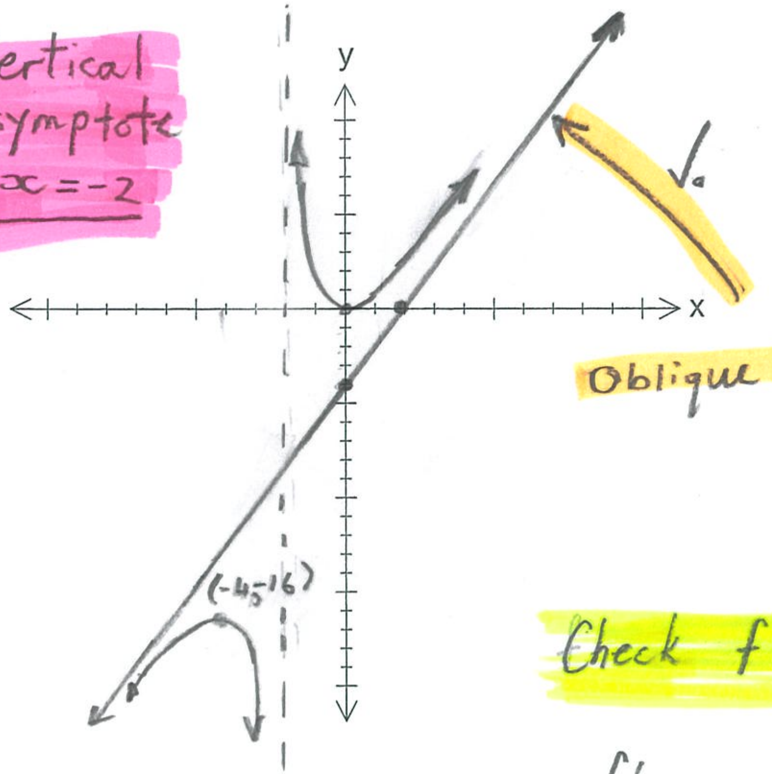
$6x^2 + 12x + 6 = 0$

$x^2 + 2x + 1 = 0$ ✓

$(x+1)^2 = 0 \Rightarrow x = -1$ ✓

Question 6 **(6 marks)**

Sketch the rational function $f(x) = \dfrac{2x^2}{x+2}$



✓. vertical asymptote $x = -2$

$$\begin{array}{r} 2x - 4 \quad ✓. \\ x+2 \overline{\smash{)}\, 2x^2 \qquad\quad} \\ -\, 2x^2 + 4x \\ \hline -4x \\ -\; -4x - 8 \\ \hline 8 \end{array}$$

✓.

Oblique Asymptote

$$f(x) = \boxed{2x-4} + \frac{8}{x+2}$$

Shape ✓.

Check $f'(x) = 0$ for max, min

$$f'(x) = \frac{(x+2)4x - 2x^2(1)}{(x+2)^2}$$

i.e $0 = \dfrac{4x^2 + 8x - 2x^2}{(x+2)^2}$ ✓.

i.e $0 = 2x^2 + 8x$

$0 = 2x(x+4)$

$\therefore x = 0$ or $x = -4$

$\Rightarrow \underline{(0,0)}$ and $\underline{(-4,-16)}$

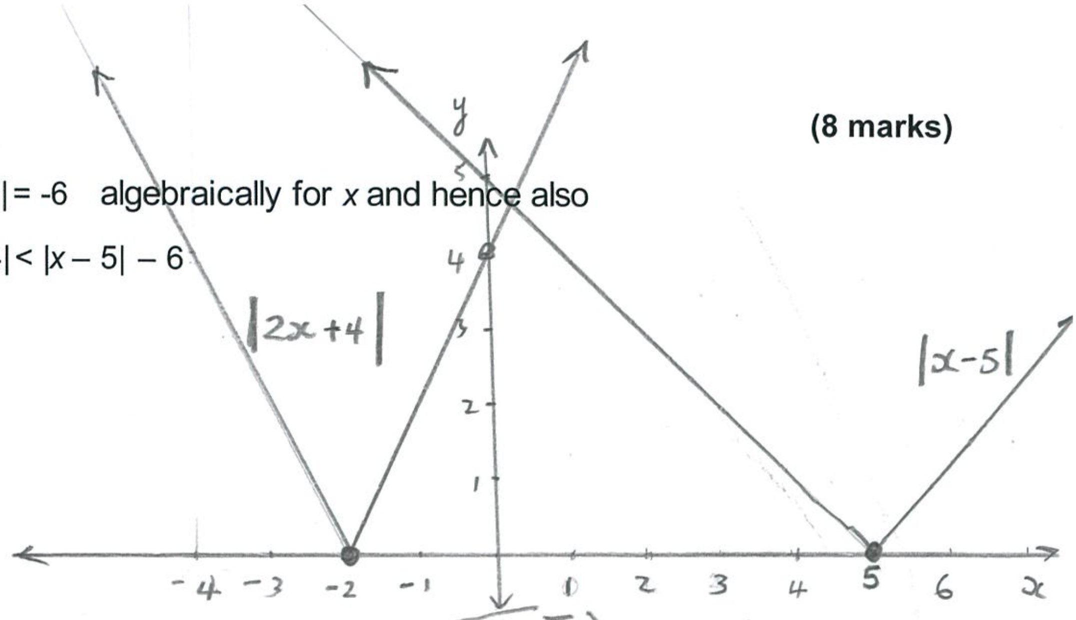
$\dfrac{2(-4)^2}{-4+2} = \dfrac{32}{-2}$

✓.

Question 7 **(8 marks)**

(i) Solve $|2x + 4| - |x - 5| = -6$ algebraically for x and hence also

(ii) Solve $|2x + 4| < |x - 5| - 6$



i) Intervals

$x < -2$ both fns negative	$-2 \le x < 5$ $\lvert 2x+4\rvert \Rightarrow$ positive, $\lvert x-5\rvert$ neg ✓.✓	$x \geq 5$ Both positive
$-(2x+4) - -(x-5) = -6$	$2x+4 - -(x-5) = -6$	$2x+4 - (x-5) = -6$
$-2x-4+x-5=-6$	$2x+4+x-5=-6$	$x+9=-6$
$-x-9=-6$	$3x-1=-6$	$x=-15$ ✓
$\boxed{x=-3}$ ✓.	$\boxed{x=\frac{-5}{3}}$ ✓.	Doesnt agree with $x \geq 5$
Which agrees with $x<-2$	which agrees with $-2 \le x < 5$	∴ Reject

(ii) $|2x+4| < |x-5| - 6$

$= |2x+4| - |x-5| < -6$ ✓.

∴ $x < -3$ agrees with $x < -2$ ✓.

and $x < -\frac{5}{3}$ agrees with $-2 \le x < 5$

∴ Solution is the Intersection

i.e $-3 < x < -\frac{5}{3}$ ✓.